MULTI-CRITERIA ANALYSIS THROUGH THE ELECTRE METHOD

Cezarina-Adina Tofan, ”Spiru Haret” University of Bucharest

Abstract: Expanding the use of the modern methods in the decision making is due, on the one hand, to the required accuracy of the process, and, on the other hand, to the introduction of the automated equipment, of the electronic computers, which require appropriate methods of working.Developing the rational decisions requires prognostication of the expected performance, the models provide means for it, and their using constitutes a scientific method for substantiating the decision. The confidence in the adopted alternatives is increased by using the mathematical modelling. Also, using the mathematical methods facilitated the possibility to optimize the multi-criteria decision.

Keywords: management methods, decisions, multi-criteria analysis.

Introduction: Decision-making process in the modern enterprise is a creative process for developing the new and values ideas. In this context, decision makers must find new methods or ways to approach the problems and at the same time to take action to use the creative methods for stimulating the creativity. These methods are widely used in the decision making process, they capitalize the creative potential of the enterprise staff, are relatively easy to apply and can be used for all types of decisions.

The decision-making process involves evaluation of the several decision variants to elect one of them. In the most cases, the evaluation of the decision-making alternatives is based on several economic indicators that are considered the criteria for evaluation.

The problems in which are searched the optimal decision variant compared with more criteria are called the multi-criteria optimization problems.

In the case of the multi-criteria optimization there is treated separately: multi-attribute and the multi-objective optimization.

The multi-criteria decisions are classified into the *multi-objective* decision, which is based on a model containing restrictions and objective functions (applying a suitable algorithm leads to a solution in relation to each objective function taken individually) and *multi-attribute* decision aimed choosing a decision variant in a finite set given, simultaneously taking into account by several criteria, which each alternative satisfies them differently[1].

Most times, for choosing the optimal decision it is required ranking of the available decisional variants relative to all desired criteria. Frequently, in practice, it is found that an optimal way in relation to a criterion is suboptimal in relation to other criteria. Then, it is chosen the variant which achieved the best compromise for all criteria.

Table 1.1 Methods of searching for "optimum multi-criteria"

Multi-objective optimization	Multi-attribute optimization
- the set of possible solutions is infinite; - optimality criteria is presented as the objective functions which must be maximized or minimized; - The solution leads to the smaller deviations towards the proposed aims through	- the set of alternatives / variants of action is finite; - each alternative is characterized by several attributes expressed quantitatively and/or qualitatively; - The chosen optimal alternative is that which satisfies all the attributes the best.

[1] Rațiu-Suciu C., *Modelare economică*, ASE Publishing House, Bucharest, 2007, page 268

the objective function.	

Processing after Rațiu-Suciu C. *Modelare economică*, ASE Publishing House, Bucharest, 2007, pg. 286.

To solve the multi-criterion problem it is used several useful methods. Such a process is based on the utility concept when it is recommended to choose the variant with the highest utility. For an easy expression in the quantity terms, it is used the utility, if the evaluation criteria are expressed in different measurement units. The composite processes are used in cases where the making decision involves the performing rankings. In view of the French school (represented by B. Roy[2]), the ELECTRE method proposes using the concordance and discordance indicators for the performance rankings.

Criteria (or attributes) are closely related to the goals and objectives; they can be different from one decision maker to another for the same decisional problem. There are cases when the evaluation criteria are of great diversity and there is an incompatibility of measurements units. In these cases, there are given the coefficients of importance to the evaluation criteria, which sum should give 1 or 100%[3]. Sometimes some criteria which are taken into consideration aim to maximize some economic indicators and other criteria to minimize some indicators.

If the multi-criteria decision-making problem is treated in terms of risk and/or uncertainty, there are specified the states of nature and probabilities of their manifestation. Different methods can lead to the different results in the multi-attribute decision. This situation is not created by the inadequacy or inappropriateness of that method, but of the decision making point of view is customized at the method level in a greater extent than in the case of the optimization algorithms.

Typical elements of a multi-attribute decisional model can be grouped in the matrix form as follows: There is V= {V1, V2 ... Vm} a lot of options and C = {CD1, CD2 ... CDn} a lot of criteria.

Decisional variants (V)	*Decisional criteria (CD)*			
	CD1	*CD2*	...	*CD3*
V1	C_{11}	C_{12}	...	C_{1n}
V2	C_{21}	C_{22}	C_{ij}	C_{2n}
...	...	...	...	...
V_m	C_{m1}	C_{m2}	...	C_{mn}

Where: V_i with i=1… m designates the set of variants in which that will be made choosing of the most suitable;

$CD_{j,}$ with j=1… n represents the set of identified criteria. An importance coefficient k_j, can be associated to each $CD_{j,}$ criterion to obtain the vector K = {k1, k2 ... kn}.

C_{ij}, with i = 1...m and j = 1...n is a numeric result that analyzes each V_i decisional variant, in terms of the criterion CD_j.

Case Study – multi-criteria analysis through the ELECTRE method based on the costs

[2] Bernard Roy (born in 1934), is an emeritus professor at the University of Paris Dauphine. In 1974 he founded "Laboratoire d'Analyse et de Modélisation des Systèmes pour l'Aide à la Décision" (LAMSADE). He worked at the graph theory and multi-criteria decision analysis (MCDA) after he had created the ELECTRE methods family. ELECTRE the acronym for "ELimination Et Choix Traduisant la REalité".

[3] Rațiu-Suciu C., *Modelare economică*, ASE Publishing House, Bucharest, 2007, pg. 269

In an engineering enterprise, it is wanted to purchase a CNC punching machine, necessary for the technological operations. To achieve this goal, he requested and obtained the information on such equipment models and delivery conditions from the various manufacturers.

The first version (**V1**) is a device that has a production capacity of 850 pieces per hour and a processing area of 8 m^2.

The second version (**V2**) is a device with a production capacity of 850 pieces per hour with semi automation to change the punches and a processing area of 8 m^2.

A third version (**V3**) has a production capacity of 4000 pieces per hour and a module which includes a storage with automation to change the punches and a performing control software and a processing area of 20 m^2.

For the three variants, which are different in terms of construction and operating principles, the manufacturers have provided the following information (Table 1.2):

Table 1.2 Information provided by the manufacturer for each variant of the installation

o	Indicators	MU	V 1	V 2	V 3
.	Purchase price	lei	71040	73320	141540
.	Processing surface	m^2	8	8	20
.	Production capacity pieces /	pieces	850	850	4000
.	The standard operational	years	25	25	20
.	Punches weight/ punches age	kg	586.3	580	1800
.	Hydraulic capacity	KN	5800	7070	34400

Based on these indicators need to identify the most advantageous variant.

The decision maker considers it is necessary, for a better evaluation of the alternatives for identification of the optimal one, to assess the following indicators:

- Specific investment (Is), calculated by reporting the purchase price at the hydraulic capacity;
- Production capacity on average per hour (Qh);
- Guaranty terms and ease of contacting a maintenance unit (Tg);
- Weight punches/punches magazine reported at the processing surface (Gp).

ELECTRE method, based on these indicators, involves the following steps:

Step 1 Decisional indicators are transformed into qualifiers, as follows:

- NS = not satisfactory;
- S = satisfactory;
- B = good;
- FB = very good.

For an easier comparison of the each other variants, it is built the table of qualifiers (Table 1.3):

Table 1.3 Table of qualifiers

Indicators	Equipment variant

	V 1	V 2	V 3
Is	S	B	FB
Qh	S	B	FB
Tg	B	S	FB
Gp	B	FB	S

Step 2 Importance coefficients (Kj) are given according to the following methodology: the indicators are placed in a table with double input (Table 1.4.). There is compared each line indicator with that from column and it will be registered at "+" if the line indicator is more important and "-" if it is less important than the column.

There are totalized "+" and are granted the coefficients of importance in direct proportion to their number.

Table 1.4 Table of the indicators comparison

i /	**Is**	**Qh**	**Tg**	**Gp**	**Total**	**Kj**
Is		+	+	+	**3**	**4**
Qh	-		-	+	**1**	**2**
Tg	-	+		+	**2**	**3**
Gp	-	-	-		**0**	**1**

Step 3 It is built a scale of notation, for each indicator, to allow the transformation of the qualifiers into the numerical size, but abstract as expression. For this, it is considered that the origin of notation scale is unsatisfactory and it is given zero mark. For other qualifying, the marks increase in direct proportion to the size of the coefficient of importance (the gap between the marks of the same indicator is equal to its coefficient of importance).

FB	-- 12	-- 6	-- 9	-- 3
B	-- 8	-- 4	-- 6	-- 2
S	-- 4	-- 2	-- 3	-- 1
NS	-- 0	-- 0	-- 0	-- 0
	Is	**Qh**	**Tg**	**Gp**

Step 4 There are calculated the coefficients of correlation, as follows:

$$C_{ij} = \frac{\sum_{s=1}^{m}\left(a_{si} - a_{sj}\right)}{\sum_{j=1}^{m} SK_j} * 100 \qquad (1.1)$$

Where: a_{si}, a_{sj} = marks form the indicators "s" and variants that compares "i" şi "j", where i>j;

Kj = coefficient of importance.

It is achieved the table of marks (Table 1.5)

Table 1.5 Table of marks

Indicators	**Equipment variant**		
	V 1	**V 2**	**V 3**

Is	**4**	**8**	**12**
Qh	**2**	**4**	**6**
Tg	**6**	**3**	**9**
Gp	**2**	**3**	**1**

Coefficients of correlation are calculated as follows:

$$c_{12} = \frac{(6-3)}{10} * 100 = 30\% \tag{1.2}$$

$$c_{13} = \frac{(2-1)}{10} * 100 = 10\% \tag{1.3}$$

$$c_{21} = \frac{(8-4)+(4-2)+(3-2)}{10} * 100 = 70\% \tag{1.4}$$

$$c_{23} = \frac{(3-1)}{10} * 100 = 20\% \tag{1.5}$$

$$c_{31} = \frac{(12-4)+(6-2)+(9-6)}{10} * 100 = 150\% \tag{1.6}$$

$$c_{32} = \frac{(12-8)+(6-4)+(9-3)}{10} * 100 = 120\% \tag{1.7}$$

The value of the correlation coefficient is given in Table 1.6:

Table 1.6 Table of the correlation coefficients value

j	**V 1**	**V 2**	**V 3**
V 1	**--**	**30**	**10**
V 2	**70**	**--**	**20**
V 3	**150**	**120**	**--**

Step 5 Coefficients of discordance are calculated as:

$$d_{ij} = \frac{\max(a_{jk} - a_{ik})}{\max N} * 100 \tag{1.8}$$

Where:

a_{jk}, a_{ik} = marks from indicators "k" where j>i, choosing the maximum difference in this comparison;

max N = the biggest difference between the extremes of all notation scales of the analyzed indicators.

There are calculated the coefficients of disagreement:

$$d_{12} = \frac{\max(8-4);(4-2);(3-2)}{12} * 100 = 33.33\% \tag{1.9}$$

$$d_{13} = \frac{\max(12-4);(6-2);(9-6)}{12} * 100 = 66.67\% \quad (1.10)$$

$$d_{21} = \frac{\max(6-3)}{12} * 100 = 25\% \quad (1.11)$$

$$d_{23} = \frac{\max(12-8);(6-4);(9-3)}{12} * 100 = 50\% \quad (1.12)$$

$$d_{31} = \frac{\max(2-1)}{12} * 100 = 8.33\% \quad (1.13)$$

$$d_{32} = \frac{\max(3-1)}{12} * 100 = 16.67\% \quad (1.14)$$

There are written the values of coefficients discrepancy in Table 1.7:

Table 1.7 Table of disagreement coefficients value

j I	V 1	V 2	V 3
V 1	--	33.33	33.3 3
V 2	25	--	50
V 3	8.3	16.67	--

Step 6 Hierarchy variants step – the stage of decision.

On this purpose there is associated a Gj graph to the variants set (manufacturers) analyzed, meaning that each vertex of the graph will be an option.

Between Vi and Vj vertices of the Gj graph, it will be drawn a bow with an arrow in Vj, if Vi is better. This conclusion is true if $c_{ij} > p$ and $d_{ij} > q$, where p and q are two thresholds (restrictions) chosen for the coefficient of correlation (p) and discordance (q). In order to have correlations between all vertices of the Gj graph, it will be reduced the p threshold and it will be increased the q threshold (not necessarily simultaneously).

p = 150 (1.15)

q = 8.3 (1.16)

Result **V 3 > V 1** (1.17)

p = 120 (1.18)

q = 16.67 (1.19)

Result **V 3 > V 2** (1.20)

p = 70 (1.21)

q = 25 (1.22)

Result **V 2 > V 1** (1.23)

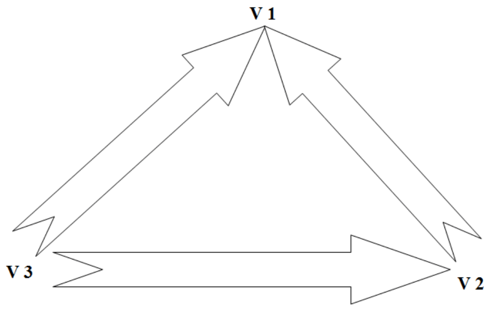

Fig. 1 Graph of the variants set

The hierarchy is as follows **V 3 > V 2 > V 1**, which results from the above figure (Figure 1). So, the equipment hypothesis **V 3** outperforms the other two hypotheses.

To take a group decision means to find a rational rule leading from the individual preference to a representative order for the entire group. It was found, however, that there is not a rule of universal rationality, but only relative rules applied in certain specific circumstances. Difficulty of the participatory decisions adoption requires consideration of the relative importance of the deciders by providing of some coefficients of importance. Under these conditions, the rationalization of the group decision can be made using one of the multi-criteria decision methods.

References:

1. Rațiu-Suciu C., *Modelare economică*, ASE Publishing House, Bucharest, 2007

2. Tofan C. A., *Data base concerning the costing management*, Economic Science Series (Annals of Spiru Haret University. Economic Sciences Series), issue: XV / 2009

3. Tofan C. A., *The management of the informatics systems projection*, Review of General Management, Brasov, vol. 10(2), 2009.

4. Tofan C. A., *Information system - a component of the management system*, Review of General Management, Spiru Haret University, Faculty of Management Brasov, vol. 17, Issue 1, 2013